Course: AUFP- FP 101

Week 2

BASIC MATHEMATICS: FP 101

by

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Complex Numbers

Objectives

- Arithmetic Operations on Complex Numbers
- Square Roots of Negative Numbers
- Complex Solutions of Quadratic Equations

Complex Numbers

Iota

We define as

$$i = \sqrt{-1}$$

$$i^2 = -1.$$

$\sqrt{-4} = \sqrt{4(-1)}$

$= \sqrt{4}\sqrt{-1}$

$= 2i$

Imaginary Numbers

Any number of the for ai is called Imaginary number, where a is real number

Exp $3i$; $-\frac{1}{5}i$

Complex Number

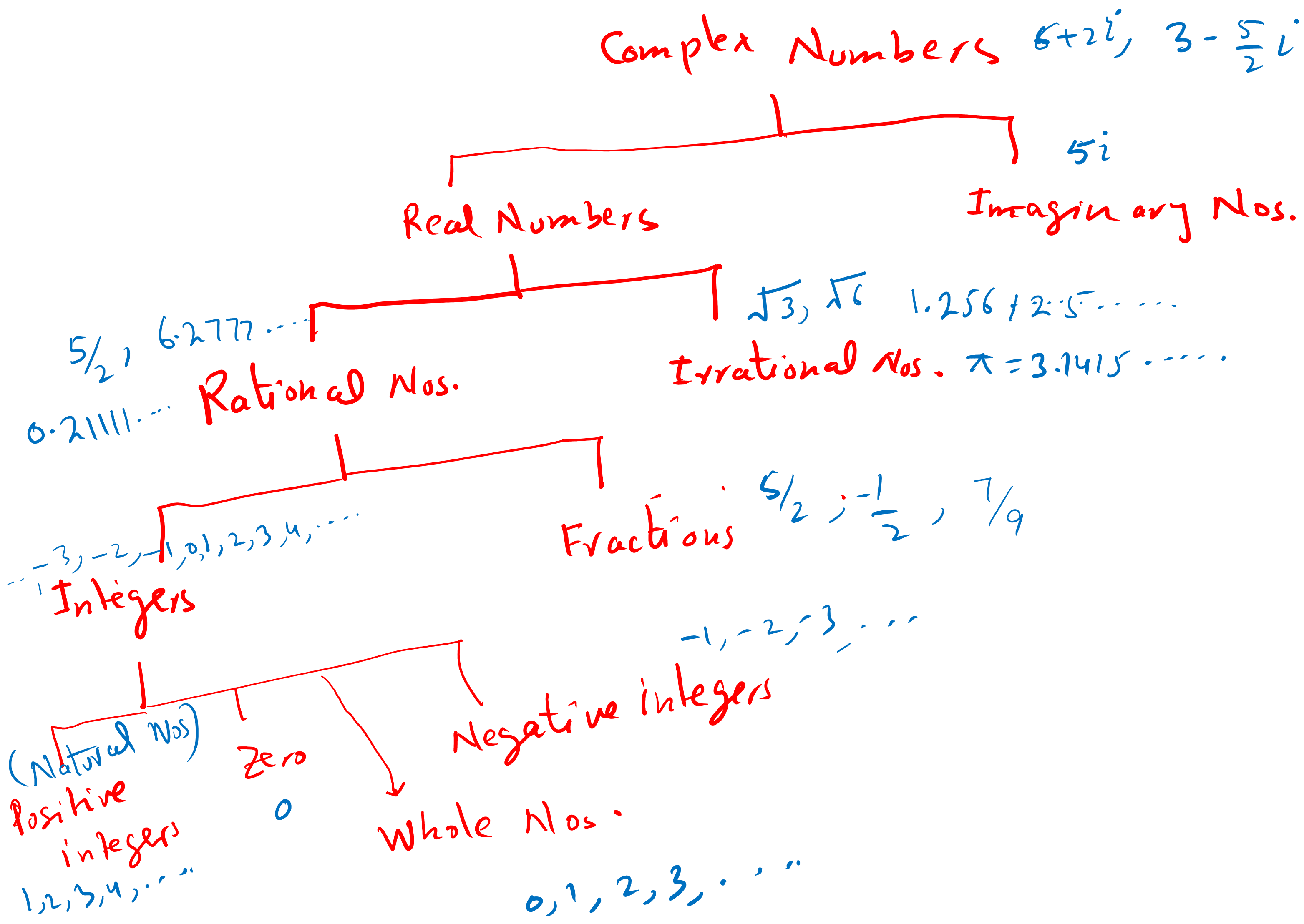
A *complex number* is a number of the form **a + bi**, where *a* and *b* are real numbers.

$a+ib = (a,b)$

e.g. $2+3i$

$5-\frac{1}{6}i$

Complex Numbers



Complex Numbers

Real and Imaginary parts of complex Numbers

$$5 + 6i$$

Real Part

Imaginary Part

Example 1 – *Complex Numbers*

The following are examples of complex numbers.

$3 + 4i$ Real part 3, imaginary part 4

$\frac{1}{2} - \frac{2}{3}i$ Real part $\frac{1}{2}$, imaginary part $-\frac{2}{3}$

$0 + 6i$ Real part 0, imaginary part 6

$-7 + 0i$ Real part –7, imaginary part 0

Complex Numbers

Solve the equation

$$x^2 + 4 = 0$$

we get $\sqrt{x^2} = \pm\sqrt{-4}$,

$$\sqrt{-4} = \sqrt{4}\,i = 2i, \qquad \sqrt{4}\sqrt{-1}$$

$$x = \pm\sqrt{-4}$$

$$= \pm 2i$$

Complex Numbers

DEFINITION OF COMPLEX NUMBERS

A **complex number** is an expression of the form

$$a + bi$$

where a and b are real numbers and $i^2 = -1$. The **real part** of this complex number is a, and the **imaginary part** is b. Two complex numbers are **equal** if and only if their real parts are equal and their imaginary parts are equal.

Note that both the real and imaginary parts of a complex number are real numbers.

Arithmetic Operations on Complex Numbers

$$\begin{array}{r} 3 + 2i \\ 6 + 7i \\ \hline \end{array}$$

$$\begin{array}{rl} & 3 + \boxed{2x} \\ (+) & 6 + \boxed{7x} \\ \hline & 9 + 9x \\ \hline \end{array}$$

Like term

$$(3 + 2x) + (6 + 7x)$$

$$= 3 + 2x + 6 + 7x$$

$$= 9 + 9x$$

Example 2 – *Adding, Subtracting, and Multiplying Complex Numbers*

Express the following in the form $a + bi$.

(a) $(3 + 5i) + (4 - 2i)$

$3 + 5i + 4 - 2i = 7 + 3i$

(b) $(3 + 5i) - (4 - 2i)$

$3 + 5i - 4 + 2i = -1 + 7i$

(c) $(3 + 5i)(4 - 2i)$

(d) i^{23}

Solution:

(a) According to the definition, we add the real parts and we add the imaginary parts.

$$(3 + 5i) + (4 - 2i) = (3 + 4) + (5 - 2)i$$

$$= 7 + 3i$$

Example 2 – *Solution*

cont'd

(b) $(3 + 5i) - (4 - 2i) = (3 - 4) + [5 - (-2)]i$

$$= -1 + 7i$$

(c) $(3 + 5i)(4 - 2i) = \cancel{[3 \cdot 4 - 5(-2)] + [3(-2) + 5 \cdot 4]i}$

$$= 22 + 14i$$

$5x(2x) = 10x^2$

$5i(2i)$
$= 10i^2$
$= 10(-1)$
$= -10$

$= 12 - 6i + 20i - 10i^2$

$= 12 + 14i - 10(-1) = 12 + 10 + 14i$

$= 12 + 10 + 14i = 22 + 14i$

Conjugate of a Complex Number

Let $z = a + bi$ *be a* complex number and its **complex conjugate** is denoted by $\bar{z}$ and defined as

$\bar{z} = \overline{a+ib}$

e.g.

$(3+2i)(3-2i)$

$$\bar{z} = a - bi.$$

$= 3^2 + 2^2 = 13$

Note that

$$z \cdot \bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

So, the product of a complex number and its conjugate is always a nonnegative real number.

Conjugate of a Complex Number

Complex Conjugates

Number	Conjugate
$3 + 2i$	$3 - 2i$
$1 - i$	$1 + i$
$4i$	$-4i$
5	5

Note
Conjugate of a
Real number
is
the same no.

Arithmetic Operations on Complex Numbers

We use this property to divide complex numbers.

$a + ib$

DIVIDING COMPLEX NUMBERS

To simplify the quotient $\dfrac{a+bi}{c+di}$, multiply the numerator and the denominator by the complex conjugate of the denominator:

$$\frac{a+bi}{c+di} = \left(\frac{a+bi}{c+di}\right)\left(\frac{c-di}{c-di}\right) = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

Example 3 – *Dividing Complex Numbers*

$-10+3=-7$

Express the following in the form *a* + *bi*.

(a) $\dfrac{3+5i}{1-2i} \cdot \dfrac{(1+2i)}{(1+2i)} = \dfrac{3+6i+5i+10i^2}{1^2+2^2} = \dfrac{3+11i+10(-1)}{5} = \dfrac{-7+11i}{5} = -\dfrac{7}{5} + \dfrac{11}{5}i$

(b) $\dfrac{7+3i}{4i}$

Solution:

We multiply both the numerator and denominator by the complex conjugate of the denominator to make the new denominator a real number.

Example 3 – *Solution*

cont'd

(a) The complex conjugate of $1 - 2i$ is $\overline{1 - 2i} = 1 + 2i$.

Therefore

$$\frac{3 + 5i}{1 - 2i} = \left(\frac{3 + 5i}{1 - 2i}\right)\left(\frac{1 + 2i}{1 + 2i}\right)$$

$$= \frac{-7 + 11i}{5}$$

$$= -\frac{7}{5} + \frac{11}{5}i$$

Example 3 – *Solution*

cont'd

(b) The complex conjugate of 4*i* is –4*i*.

Therefore

$$\frac{7 + 3i}{4i} = \left(\frac{7 + 3i}{4i}\right)\left(\frac{-4i}{-4i}\right)$$

$$= \frac{12 - 28i}{16}$$

$$= \frac{3}{4} - \frac{7}{4}i$$

Square Roots of Negative Numbers

Square Roots of Negative Numbers

SQUARE ROOTS OF NEGATIVE NUMBERS

If $-r$ is negative, then the **principal square root** of $-r$ is

$$\sqrt{-r} = i\sqrt{r}$$

The two square roots of $-r$ are $i\sqrt{r}$ and $-i\sqrt{r}$.

Example 4 – *Square Roots of Negative Numbers*

(a) $\sqrt{-1} = i\sqrt{1}$

$= i$

(b) $\sqrt{-16} = i\sqrt{16}$

$= \underline{4i}$

or $\sqrt{3}\, i$ or $i\sqrt{3}$

(c) $\sqrt{-3} = i\sqrt{3}$

Square Roots of Negative Numbers

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Although $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ when *a* and *b* are positive, this is *not* true when both are negative.

$\sqrt{2} \cdot \sqrt{3} = \sqrt{2 \cdot 3} = \sqrt{6}$

$\sqrt{6} \cdot \sqrt{7} = \sqrt{6 \cdot 7} = \sqrt{42}$

$\sqrt{-2}\,\sqrt{-3} =$

$\cancel{\sqrt{(-2)(-3)} = \sqrt{6}}$

Not Allowed.

$\sqrt{2}\,i \quad \sqrt{3}\,i = \sqrt{2} \cdot \sqrt{3}\; i^2$

$= \sqrt{2 \cdot 3}\; i^2$

$= \sqrt{6}\,(-1) = -\sqrt{6}$

Square Roots of Negative Numbers

For example,

$$\sqrt{-2}\cdot\sqrt{-3} = i\sqrt{2}\cdot i\sqrt{3} = i^2\sqrt{6} = -\sqrt{6}$$

but $$\sqrt{(-2)(-3)} = \sqrt{6}$$

so $$\sqrt{-2}\cdot\sqrt{-3} \neq \sqrt{(-2)(-3)}$$

Complex Numbers

SKILLS

7–16 ■ Real and Imaginary Parts Find the real and imaginary parts of the complex number.

Answers

7. $5 - 7i$

9. $\dfrac{-2 - 5i}{3}$

11. 3

13. $-\frac{2}{3}i$

15. $\sqrt{3} + \sqrt{-4}$

7. $5 - 7i$: real part 5, imaginary part -7.

9. $\dfrac{-2 - 5i}{3} = -\frac{2}{3} - \frac{5}{3}i$: real part $-\frac{2}{3}$, imaginary part $-\frac{5}{3}$.

11. 3: real part 3, imaginary part 0.

13. $-\frac{2}{3}i$: real part 0, imaginary part $-\frac{2}{3}$.

15. $\sqrt{3} + \sqrt{-4} = \sqrt{3} + 2i$: real part $\sqrt{3}$, imaginary part 2.

Complex Numbers

27–36 ■ Products Evaluate the product, and write the result in the form $a + bi$.

27. $4(-1 + 2i)$

29. $(7 - i)(4 + 2i)$

31. $(6 + 5i)(2 - 3i)$

33. $(2 + 5i)(2 - 5i)$

35. $(2 + 5i)^2$

Complex Numbers

Answers

27. $4(-1+2i) = -4+8i$

29. $(7-i)(4+2i) = 28+14i-4i-2i^2 = (28+2)+(14-4)i = 30+10i$

31. $(6+5i)(2-3i) = 12-18i+10i-15i^2 = (12+15)+(-18+10)i = 27-8i$

33. $(2+5i)(2-5i) = 2^2-(5i)^2 = 4-25(-1) = 29$

35. $(2+5i)^2 = 2^2+(5i)^2+2(2)(5i) = 4-25+20i = -21+20i$

Complex Numbers

37–46 ■ Quotients Evaluate the quotient, and write the result in the form $a + bi$.

37. $\dfrac{1}{i}$

39. $\dfrac{2 - 3i}{1 - 2i}$

41. $\dfrac{10i}{1 - 2i}$

43. $\dfrac{4 + 6i}{3i}$

45. $\dfrac{1}{1 + i} - \dfrac{1}{1 - i}$

Answers

37. $\frac{1}{i}=\frac{1}{i}\cdot\frac{i}{i}=\frac{i}{i^2}=\frac{i}{-1}=-i$

39. $\frac{2-3i}{1-2i}=\frac{2-3i}{1-2i}\cdot\frac{1+2i}{1+2i}=\frac{2+4i-3i-6i^2}{1-4i^2}=\frac{(2+6)+(4-3)\,i}{1+4}=\frac{8+i}{5}$ or $\frac{8}{5}+\frac{1}{5}i$

41. $\frac{10i}{1-2i}=\frac{10i}{1-2i}\cdot\frac{1+2i}{1+2i}=\frac{10i+20i^2}{1-4i^2}=\frac{-20+10i}{1+4}=\frac{5\,(-4+2i)}{5}=-4+2i$

43. $\frac{4+6i}{3i}=\frac{4+6i}{3i}\cdot\frac{3i}{3i}=\frac{12i+18i^2}{9i^2}=\frac{-18+12i}{-9}=\frac{-18}{-9}+\frac{12}{-9}i=2-\frac{4}{3}i$

45. $\frac{1}{1+i}-\frac{1}{1-i}=\frac{1}{1+i}\cdot\frac{1-i}{1-i}-\frac{1}{1-i}\cdot\frac{1+i}{1+i}=\frac{1-i}{1-i^2}-\frac{1+i}{1-i^2}=\frac{1-i}{2}+\frac{-1-i}{2}=-i$

53–60 ■ Radical Expressions Evaluate the radical expression, and express the result in the form $a + bi$.

53. $\sqrt{-49}$

55. $\sqrt{-3}\sqrt{-12}$

59. $\dfrac{2 + \sqrt{-8}}{1 + \sqrt{-2}}$

Answers

53. $\sqrt{-49} = \sqrt{49}\sqrt{-1} = 7i$

55. $\sqrt{-3}\sqrt{-12} = i\sqrt{3} \cdot 2i\sqrt{3} = 6i^2 = -6$

59. $\dfrac{2+\sqrt{-8}}{1+\sqrt{-2}} = \dfrac{2+2i\sqrt{2}}{1+i\sqrt{2}} = \dfrac{2\left(1+i\sqrt{2}\right)}{1+i\sqrt{2}} = 2$


Thank you