Course: AUFP- FP 101 **Week 2**

BASIC MATHEMATICS: **FP 101**

by

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The Coordinate Plane;
Graphs of Equations; Circles

Objectives

- The Coordinate Plane
- The Distance and Midpoint Formulas
- Graphs of Equations in Two Variables
- Intercepts
- Circles
- Symmetry

The Coordinate Plane

The Coordinate Plane

Just as points on a line can be identified with real numbers to form the coordinate line, points in a plane can be identified with ordered pairs of numbers to form the **coordinate plane** or **Cartesian plane**.

To do this, we draw two perpendicular real lines that intersect at 0 on each line.

Usually, one line is horizontal with positive direction to the right and is called the ***x*-axis**; the other line is vertical with positive direction upward and is called the ***y*-axis**.

The Coordinate Plane

The point of intersection of the *x*-axis and the *y*-axis is the **origin *O***, and the two axes divide the plane into four **quadrants**, labeled I, II, III, and IV in Figure 1. (The points *on* the coordinate axes are not assigned to any quadrant.)

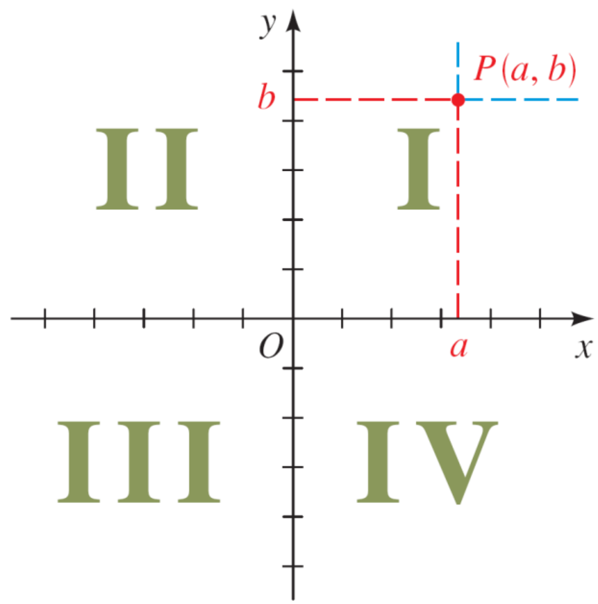


Figure 1

The Coordinate Plane

Any point *P* in the coordinate plane can be located by a unique **ordered pair** of numbers (*a*, *b*), as shown in Figure 1.

The first number *a* is called the ***x*-coordinate** of *P*; the second number *b* is called the ***y*-coordinate** of *P*.

The Coordinate Plane

We can think of the coordinates of *P* as its "address," because they specify its location in the plane. Several points are labeled with their coordinates in Figure 2.

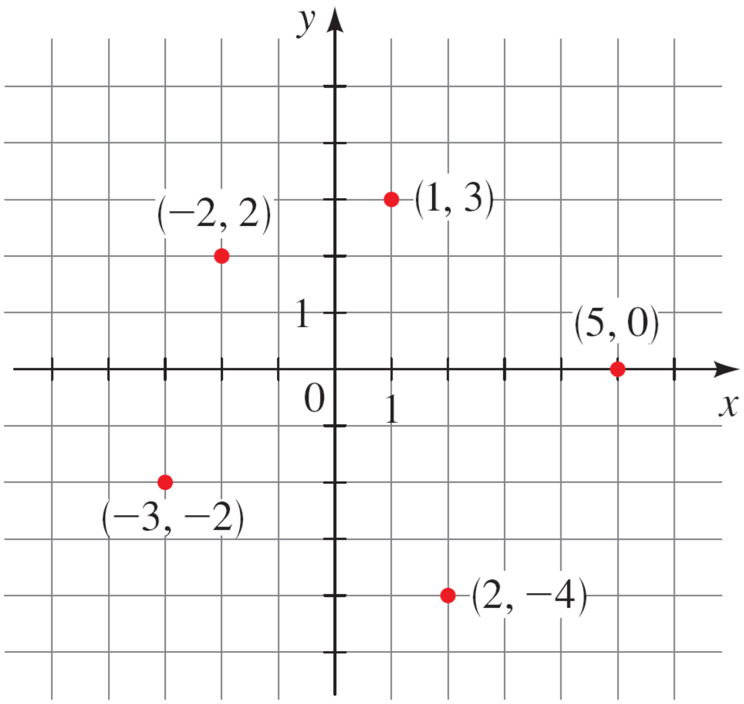


Figure 2

Example 1 – *Graphing Regions in the Coordinate Plane*

Describe and sketch the regions given by each set.

(a) $\{(x, y) \mid x \geq 0\}$

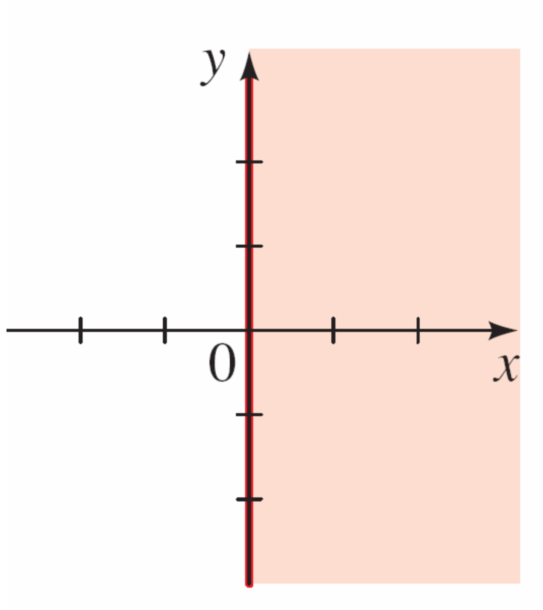
(b) $\{(x, y) \mid y = 1\}$

(c) $\{(x, y) \mid |y| < 1\}$

Example 1(a) – *Solution*

cont'd

The points whose *x*-coordinates are 0 or positive lie on the *y*-axis or to the right of it, as shown in Figure 3(a).



$x \geq 0$

Figure 3(a)

Example 1(b) – *Solution*

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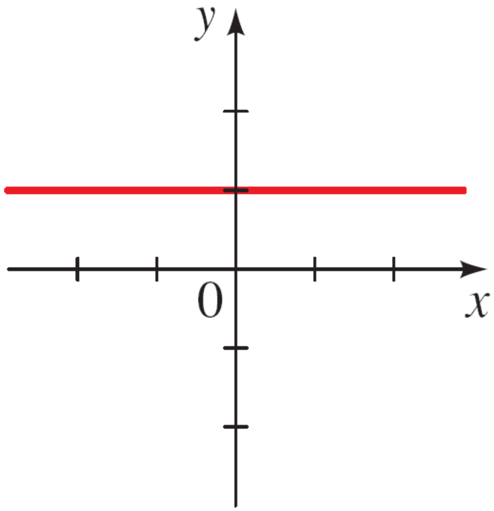
The set of all points with *y*-coordinate 1 is a horizontal line one unit above the *x*-axis, as shown in Figure 3(b).



$y = 1$

Figure 3(b)

Example 1(c) – *Solution*

cont'd

We know that

$$|y| < 1 \qquad \text{if and only if} \qquad -1 < y < 1$$

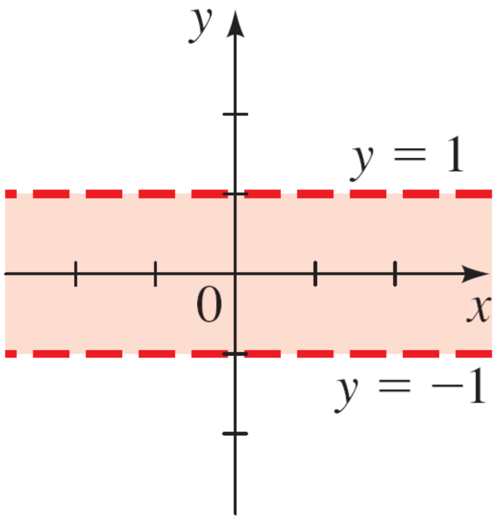
So the given region consists of those points in the plane whose y-coordinates lie between –1 and 1.

Thus the region consists of all points that lie between (but not on) the horizontal lines $y = 1$ and $y = -1$.

Example 1(c) – *Solution*

cont'd

These lines are shown as broken lines in Figure 3(c) to indicate that the points on these lines are not in the set.



$|y| < 1$

Figure 3(c)

The Distance and Midpoint Formulas

The Distance and Midpoint Formulas

We now find a formula for the distance $d(A, B)$ between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane.

We know that the distance between points a and b on a number line is $d(a, b) = |b - a|$.

The Distance and Midpoint Formulas

So from Figure 4 we see that the distance between the points $A(x_1, y_1)$ and $C(x_2, y_1)$ on a horizontal line must be $|x_2 - x_1|$, and the distance between $B(x_2, y_2)$ and $C(x_2, y_1)$ on a vertical line must be $|y_2 - y_1|$.

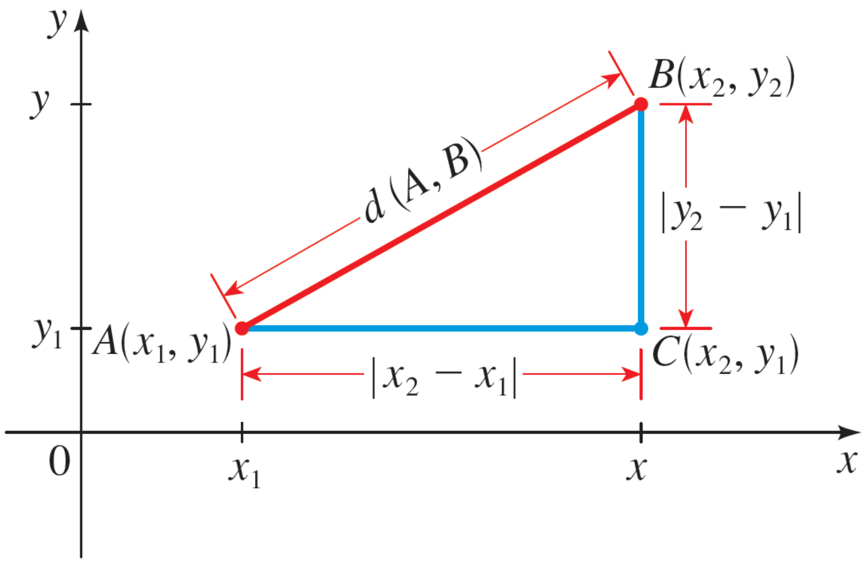


Figure 4

The Distance and Midpoint Formulas

Since triangle *ABC* is a right triangle, the Pythagorean Theorem gives

$$d(A, B) = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

DISTANCE FORMULA

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2 – *Applying the Distance Formula*

Which of the points $P(1, –2)$ or $Q(8, 9)$ is closer to the point $A(5, 3)$?

Solution:

By the Distance Formula we have

$$d(P, A) = \sqrt{(5 - 1)^2 + [3 - (-2)]^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$$

$$d(Q, A) = \sqrt{(5 - 8)^2 + (3 - 9)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45}$$

Example 2 – *Solution*

cont'd

This shows that $d(P, A) < d(Q, A)$, so P is closer to A (see Figure 5).

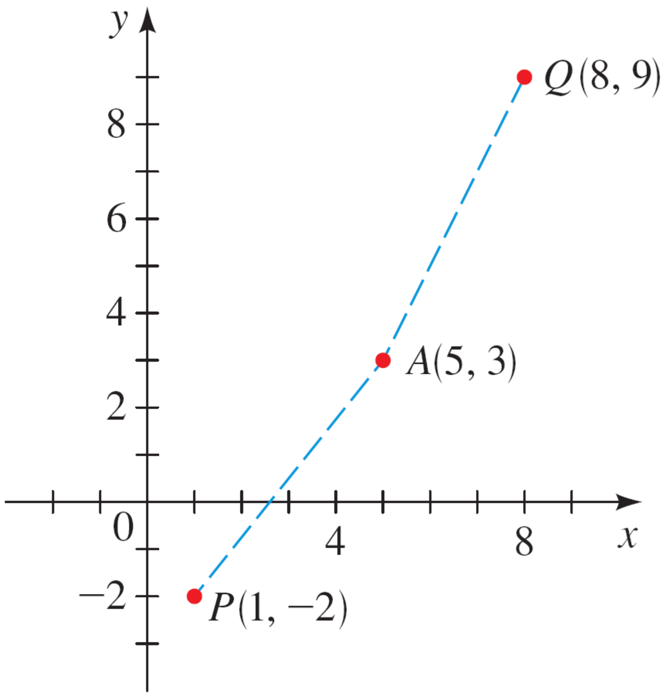


Figure 5

The Distance and Midpoint Formulas

MIDPOINT FORMULA

The midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 3 – *Applying the Midpoint Formula*

Show that the quadrilateral with vertices *P*(1, 2), *Q*(4, 4), *R*(5, 9), and *S*(2, 7) is a parallelogram by proving that its two diagonals bisect each other.

Solution:

If the two diagonals have the same midpoint, then they must bisect each other.

The midpoint of the diagonal *PR* is

$$\left(\frac{1+5}{2}, \frac{2+9}{2}\right) = \left(3, \frac{11}{2}\right)$$

Graphs of Equations in Two Variables

Graphs of Equations in Two Variables

THE GRAPH OF AN EQUATION

The **graph** of an equation in x and y is the set of all points (x, y) in the coordinate plane that satisfy the equation.

Example 4 – *Sketching a Graph by Plotting Points*

Sketch the graph of the equation $2x - y = 3$.

Solution:

We first solve the given equation for y to get

$$y = 2x - 3$$

This helps us calculate the y-coordinates in the following table.

x	$y = 2x - 3$	(x, y)
-1	-5	$(-1, -5)$
0	-3	$(0, -3)$
1	-1	$(1, -1)$
2	1	$(2, 1)$
3	3	$(3, 3)$
4	5	$(4, 5)$

Example 4 – *Solution*

cont'd

We plot the points we found in Figure 8; they appear to lie on a line. So we complete the graph by joining the points by a line.

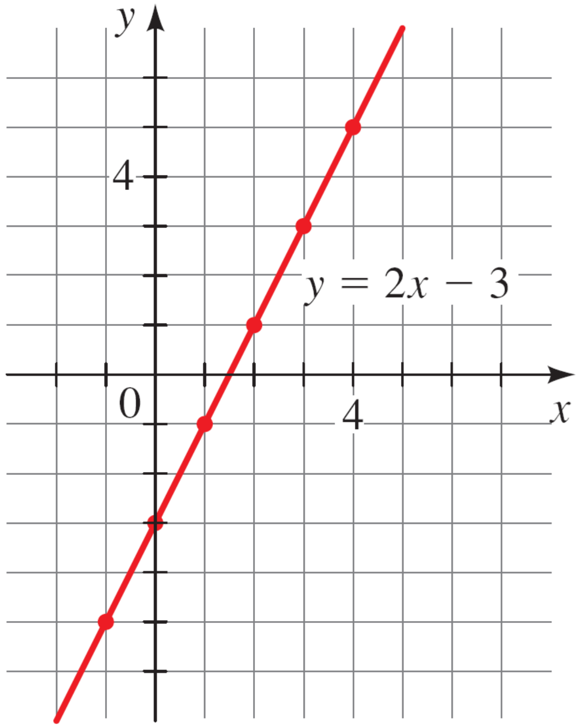
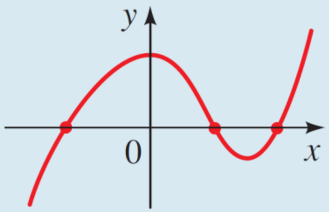
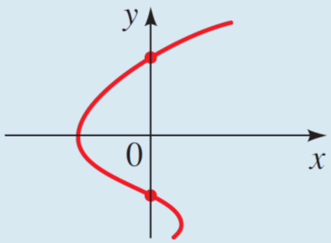


Figure 8

Intercepts

Intercepts

DEFINITION OF INTERCEPTS

Intercepts	How to find them	Where they are on the graph
***x*-intercepts:** The x-coordinates of points where the graph of an equation intersects the x-axis	Set $y = 0$ and solve for x	
***y*-intercepts:** The y-coordinates of points where the graph of an equation intersects the y-axis	Set $x = 0$ and solve for y	

Example 8 – *Finding Intercepts*

Find the x- and y-intercepts of the graph of the equation $y = x^2 - 2$.

Solution:

To find the x-intercepts, we set $y = 0$ and solve for x. Thus

$$0 = x^2 - 2 \qquad \text{Set } y = 0$$

$$x^2 = 2 \qquad \text{Add 2 to each side}$$

$$x = \pm\sqrt{2} \qquad \text{Take the square root}$$

The x-intercepts are $\sqrt{2}$ and $-\sqrt{2}$.

Example 8 – *Solution*

cont'd

To find the y-intercepts, we set $x = 0$ and solve for y. Thus

$$y = 0^2 - 2 \qquad \text{Set } x = 0$$

$$y = -2$$

The y-intercept is –2.

Example 8 – *Solution*

cont'd

The graph of this equation is sketched in Figure 12 with the *x*- and *y*-intercepts labeled.

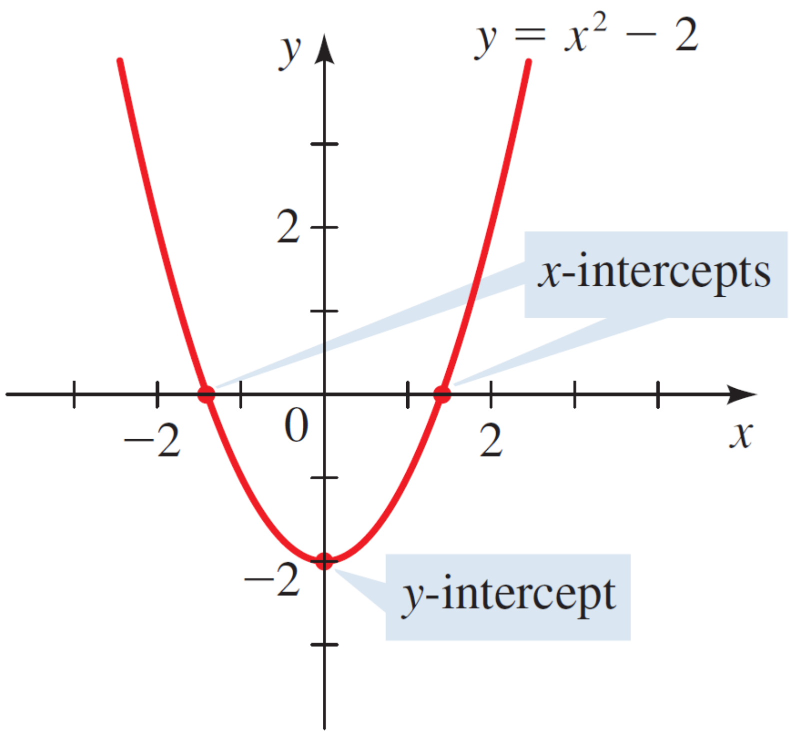


Figure 12

Circles

Circles

EQUATION OF A CIRCLE

An equation of the circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

This is called the **standard form** for the equation of the circle. If the center of the circle is the origin $(0, 0)$, then the equation is

$$x^2 + y^2 = r^2$$

Example 10 – *Finding an Equation of a Circle*

(a) Find an equation of the circle with radius 3 and center (2, –5).

(b) Find an equation of the circle that has the points $P(1, 8)$ and $Q(5, -6)$ as the endpoints of a diameter.

Solution:

(a) Using the equation of a circle with $r = 3$, $h = 2$, and $k = -5$, we obtain

$$(x - 2)^2 + (y + 5)^2 = 9$$

Example 10 – *Solution*

cont'd

The graph is shown in Figure 16.

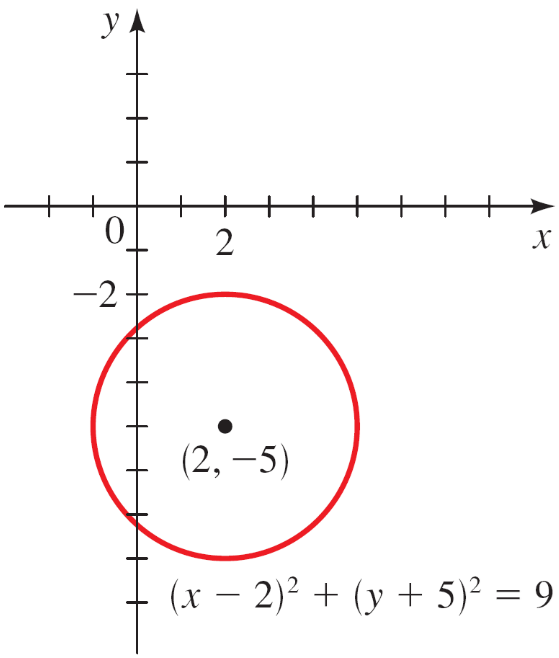


Figure 16

Example 10 – *Solution*

cont'd

(b) We first observe that the center is the midpoint of the diameter *PQ*, so by the Midpoint Formula the center is

$$\left(\frac{1+5}{2}, \frac{8-6}{2}\right) = (3, 1)$$

The radius *r* is the distance from *P* to the center, so by the Distance Formula

$$r^2 = (3 - 1)^2 + (1 - 8)^2 = 2^2 + (-7)^2 = 53$$

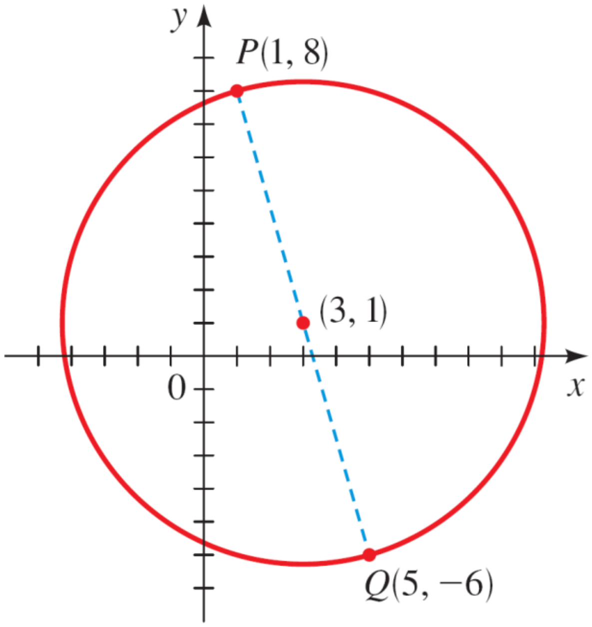
Therefore the equation of the circle is

$$(x - 3)^2 + (y - 1)^2 = 53$$

Example 10 – *Solution*

cont’d

The graph is shown in Figure 17.



$(x - 3)^2 + (y - 1)^2 = 53$

Figure 17

Symmetry

Symmetry

Figure 18 shows the graph of $y = x^2$. Notice that the part of the graph to the left of the y-axis is the mirror image of the part to the right of the y-axis.

The reason is that if the point (x, y) is on the graph, then so is $(-x, y)$, and these points are reflections of each other about the y-axis.

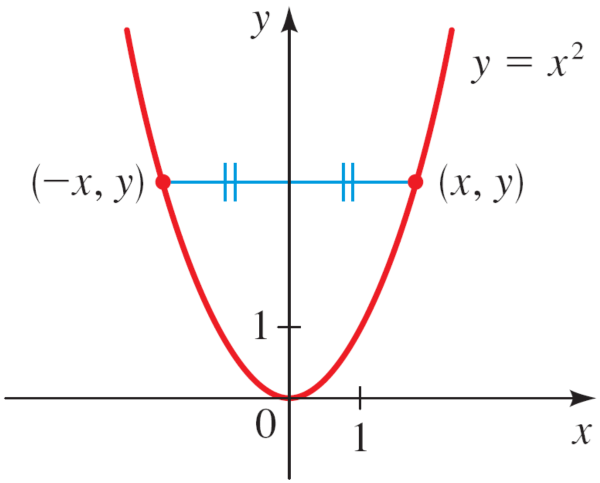


Figure 18

Symmetry

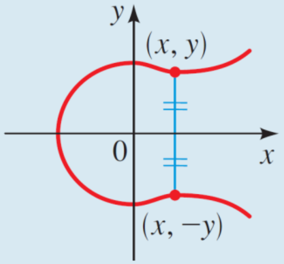
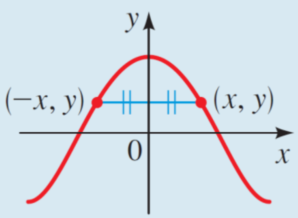
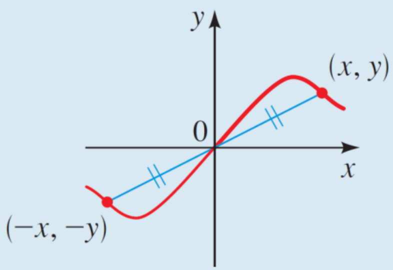
In this situation we say that the graph is **symmetric with respect to the *y*-axis**.

Similarly, we say that a graph is **symmetric with respect to the *x*-axis** if whenever the point (x, y) is on the graph, then so is (x, $-y$).

A graph is **symmetric with respect to the origin** if whenever (x, y) is on the graph, so is ($-x$, $-y$). (We often say symmetric “about” instead of “with respect to.”)

Symmetry

TYPES OF SYMMETRY

Symmetry	Test	Graph	Property of Graph
With respect to the x-axis	Replace y by $-y$. The resulting equation is equivalent to the original one.		Graph is unchanged when reflected about the x-axis. See Figures 14 and 19.
With respect to the y-axis	Replace x by $-x$. The resulting equation is equivalent to the original one.		Graph is unchanged when reflected about the y-axis. See Figures 9, 10, 11, 12, 14, and 18.
With respect to the origin	Replace x by $-x$ and y by $-y$. The resulting equation is equivalent to the original one.		Graph is unchanged when rotated 180° about the origin. See Figures 14 and 20.


Thank you