Course: AUFP- FP 101

Week 2

BASIC MATHEMATICS: FP 101

by

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Lines

Objectives

- The Slope of a Line
- Point-Slope Form of the Equation of a Line
- Slope-Intercept Form of the Equation of a Line
- Vertical and Horizontal Lines
- General Equation of a Line
- Parallel and Perpendicular Lines

The Slope of a Line

The Slope of a Line

The *slope* of a line is the ratio of rise to run:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

If a line lies in a coordinate plane, then the **run** is the change in the *x*-coordinate and the **rise** is the corresponding change in the *y*-coordinate between any two points on the line (see Figure 2).

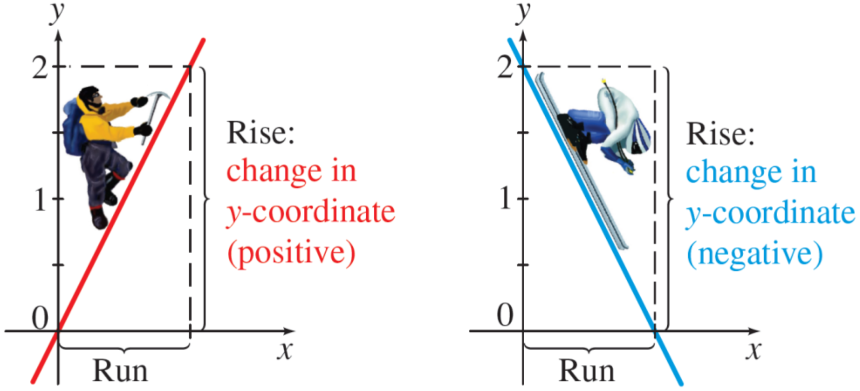


Figure 2

The Slope of a Line

This gives us the following definition of slope.

SLOPE OF A LINE

The **slope** m of a nonvertical line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is not defined.

The slope is independent of which two points are chosen on the line.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2' - y_1'}{x_2' - x_1'}$$

The Slope of a Line

We can see that this is true from the similar triangles in Figure 3.

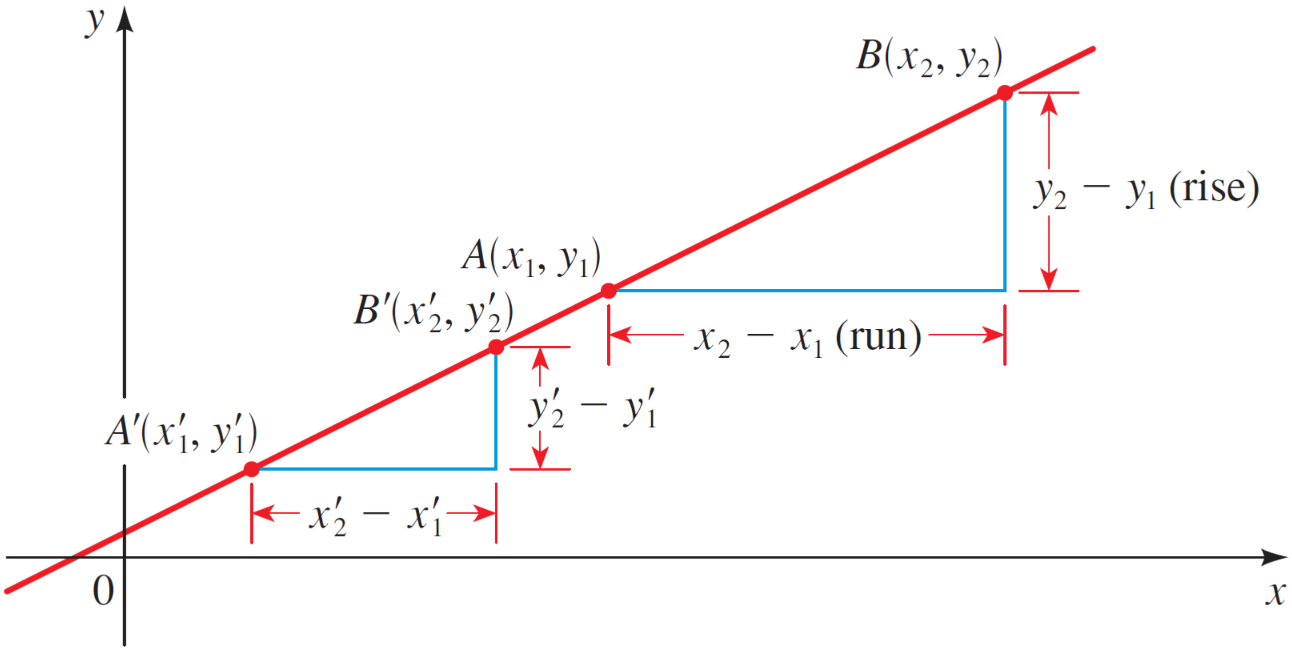


Figure 3

Example 1 – *Finding the Slope of a Line Through Two Points*

Find the slope of the line that passes through the points *P*(2, 1) and *Q*(8, 5).

Solution:

Since any two different points determine a line, only one line passes through these two points. From the definition the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{8 - 2} = \frac{4}{6} = \frac{2}{3}$$

This says that for every 3 units we move to the right, the line rises 2 units.

Example 1 – *Solution*

cont'd

The line is drawn in Figure 4.

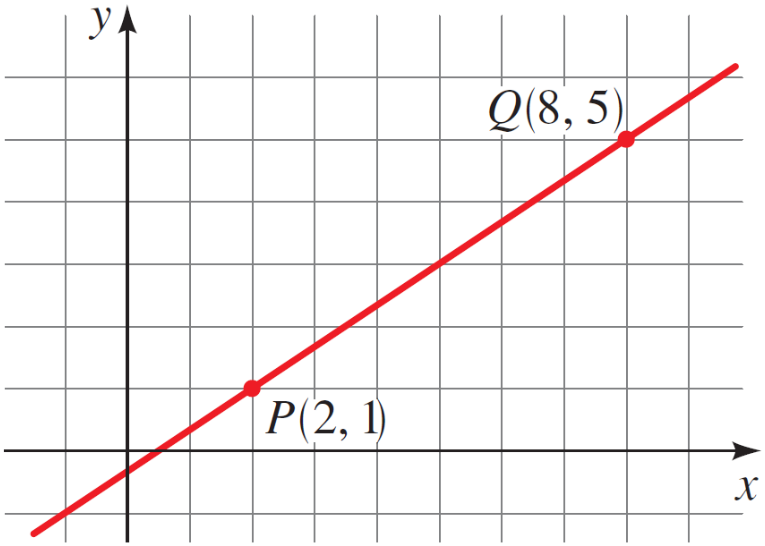
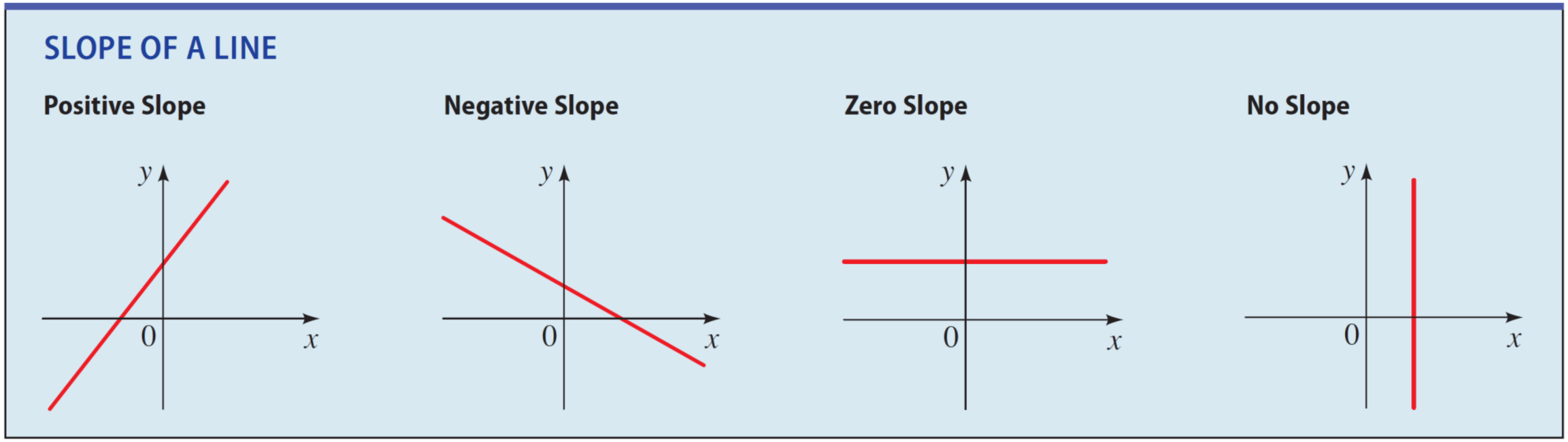


Figure 4

The Slope of a Line

The figures in the box below show several lines labeled with their slopes.



Point-Slope Form of the Equation of a Line

Point-Slope Form of the Equation of a Line

POINT-SLOPE FORM OF THE EQUATION OF A LINE

An equation of the line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1)$$

Example 2 – *Finding an Equation of a Line with Given Point and Slope*

(a) Find an equation of the line through (1, –3) with slope $-\frac{1}{2}$.

(b) Sketch the line.

Solution:

(a) Using the point-slope form with $m = -\frac{1}{2}$, $x_1 = 1$, and $y_1 = -3$, we obtain an equation of the line as

$$y + 3 = -\frac{1}{2}(x - 1)$$ Slope $m = -\frac{1}{2}$, point (1, –3)

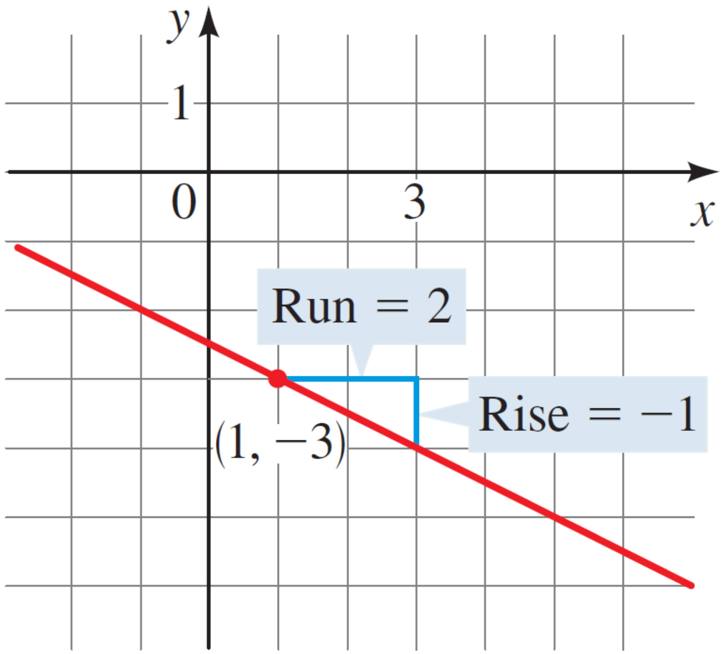
$$2y + 6 = -x + 1$$ Multiply by 2

$$x + 2y + 5 = 0$$ Rearrange

Example 2 – *Solution*

cont'd

(b) $x + 2y + 5 = 0$



Slope-Intercept Form of the Equation of a Line

Slope-Intercept Form of the Equation of a Line

Suppose a nonvertical line has slope *m* and *y*-intercept *b* (see Figure 7).

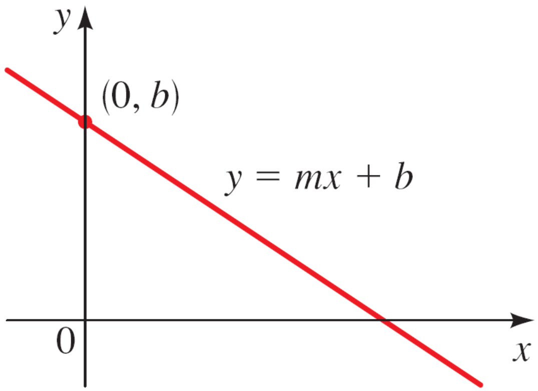


Figure 7

This means that the line intersects the *y*-axis at the point (0, *b*), so the point-slope form of the equation of the line, with $x = 0$ and $y = b$, becomes

$$y - b = m(x - 0)$$

Slope-Intercept Form of the Equation of a Line

This simplifies to $y = mx + b$, which is called the **slope-intercept form** of the equation of a line.

SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

An equation of the line that has slope m and y-intercept b is

$$y = mx + b$$

Example 4 – *Lines in Slope-Intercept Form*

(a) Find an equation of the line with slope 3 and y-intercept –2.

(b) Find the slope and y-intercept of the line $3y - 2x = 1$.

Solution:

(a) Since $m = 3$ and $b = -2$,

$$y = 3x - 2$$

Example 4 – *Solution*

cont'd

(b) We first write the equation in the form ***y* = *mx* + *b*.**

$$3y - 2x = 1$$

$$3y = 2x + 1$$

$$y = \frac{2}{3}x + \frac{1}{3}$$

Which is ***slope-intercept form*** of the equation of a line,

slope is $m = \frac{2}{3}$

y-intercept is $b = \frac{1}{3}$.

Vertical and Horizontal Lines

Vertical and Horizontal Lines

VERTICAL AND HORIZONTAL LINES

- An equation of the vertical line through (a, b) is $x = a$.
- An equation of the horizontal line through (a, b) is $y = b$.

Example 5 – *Vertical and Horizontal Lines*

(a) An equation for the vertical line through (3, 5) is $x = 3$.

(b) The graph of the equation $x = 3$ is a vertical line with x-intercept 3.

(c) An equation for the horizontal line through (8, –2) is $y = -2$.

(d) The graph of the equation $y = -2$ is a horizontal line with y-intercept –2.

Example 5 – *Vertical and Horizontal Lines* cont'd

The lines are graphed in Figure 9.

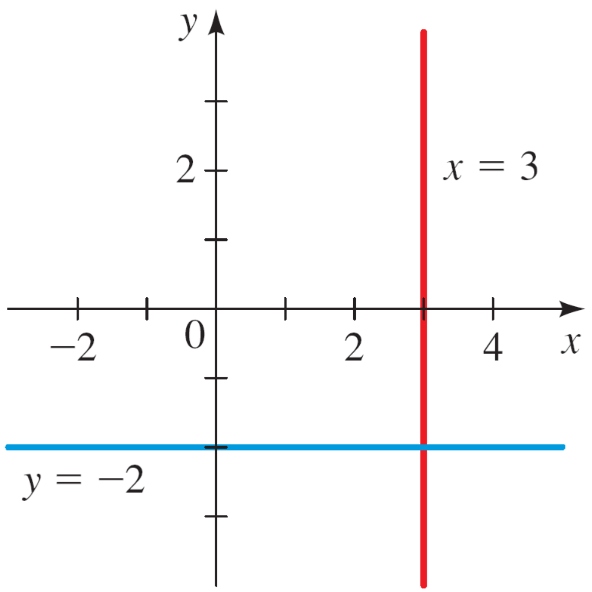


Figure 9

General Equation of a Line

General Equation of a Line

A **linear equation** in the variables x and y is an equation of the form

$$Ax + By + C = 0$$

where A, B, and C are constants and A and B are not both 0.

Equation of a line In **slope Intercept Form**

An equation of a line is a linear equation:

- A nonvertical line has the equation $y = mx + b$

General Equation of a Line

GENERAL EQUATION OF A LINE

The graph of every **linear equation**

$$Ax + By + C = 0 \qquad (A, B \text{ not both zero})$$

is a line. Conversely, every line is the graph of a linear equation.

Example 6 – *Graphing a Linear Equation*

Sketch the graph of the equation $2x - 3y - 12 = 0$.

Solution 1:

x-intercept:

Substitute $y = 0$,

$$2x - 12 = 0,$$
$$x = 6$$

y-intercept:

Substitute $x = 0$,

$$-3y - 12 = 0,$$
$$y = -4$$

Example 6 – *Solution*

cont'd

Graph

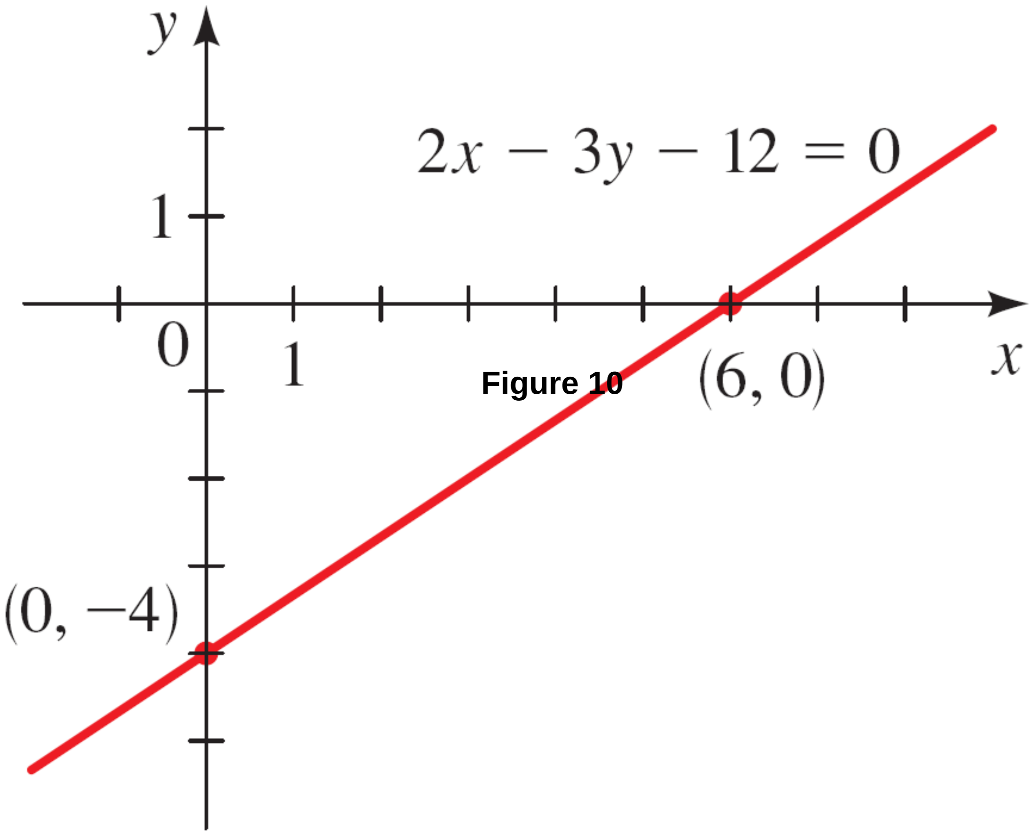


Figure 10

Example 6 – *Solution*

cont'd

write the **equation in slope-intercept form**.

$$2x - 3y - 12 = 0$$

Solution:

$$2x - 3y = 12$$

$$-3y = -2x + 12$$

$$y = \quad x - 4$$

This equation is in the form $y = mx + b$,

so the slope is m =1
and the y-intercept is $b = -4$.

Parallel and Perpendicular Lines

Parallel and Perpendicular Lines

PARALLEL LINES

Two nonvertical lines are parallel if and only if they have the same slope.

Example 7 – *Finding an Equation of a Line Parallel to a Given Line*

Find an equation of the line through the point (5, 2) that is parallel to the line $4x + 6y + 5 = 0$.

Solution:

Example 7 – *Solution*

cont'd

Thus an equation of the required line is $2x + 3y - 16 = 0$.

Parallel and Perpendicular Lines

PERPENDICULAR LINES

Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$, that is, their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

Also, a horizontal line (slope 0) is perpendicular to a vertical line (no slope).

Example 8 – *Perpendicular Lines*

Show that the points *P*(3, 3), *Q*(8, 17), and *R*(11, 5) are the vertices of a right triangle.

Solution:

The slopes of the lines containing *PR* and *QR* are, respectively,

$$m_1 = \frac{5 - 3}{11 - 3} = \frac{1}{4} \quad \text{and} \quad m_2 = \frac{5 - 17}{11 - 8} = -4$$

Since $m_1 m_2 = (1/4)(-4) = -1$,

these lines are perpendicular, so *PQR* is a right triangle.

Example 8 – *Solution*

cont'd

Question#2

Find an equation of the line that satisfies the given conditions.
Through $(-3, 1)$; perpendicular to the line $y = \left(-\frac{1}{2}\right)x + 8$

$$y = 2x + 7$$

Example 8 – *Solution*

cont'd

Question#3

Find an equation of the line that satisfies the given conditions. Through (6, 7); parallel to the line passing through (7, 5) and (3, 1)


Thank you